Spatial Distribution in a Red Pine - Balsam Fir Stand, Itasca State Park, Minnesota*

Larry A. Drew

May, 1970

* Presented to the Lake Itasca Forestry and Biology Station, University of Minnesota, as fulfillment of contract agreement for NSF funding for summer research.

Table of Contents

I. Introduction and Acknowledgments

On the west side of Lake Itasca, in the Wilderness Sanctuary portion of Itasca State Park, Minnesota (SW¼, SE¼, Sec. 16, T142N, R36W) there exists a set of permanent sample points which have been in existence since 1953. These points were established by Dr. Egolfs V. Bakuzis of the School of Forestry, University of Minnesota. Records of the stand history have been kept by Dr. Bakuzis; a more permanent record and a better understanding of the distributions within the stand were desirable and thus it was conceived that a map of the stand itself would be a valuable asset.

A mapping of a portion of the stand and an analysis of the pattern present was the scope of the present study. This study was supported by a National Science Foundation grant through the Itasca Forestry and Biology Station, University of Minnesota, and by the School of Forestry, University of Minnesota. The mapping procedure was performed during the late summer of 1967 and map construction and data analysis completed during the winters of subsequent years to 1970.

Contained in this report is a copy of the map used for the analysis of pattern in this study; reproduction of the map for education or research purposes is permitted by consent of the Director of Field Biology, Itasca Forestry and Biology Station, or from Dr. Egolfs V. Bakuzis, School of Forestry, University of Minnesota.

II. Methods

There have been studies reported in which a stand of trees was mapped and the resulting map used in sampling. In particular, two studies have been conducted in the Itasca area (Ohmann, 1968; Sudia, 1960); these studies were done in different types than the current study (jack pine (Pinus banksiana) and maple (Acer saccharum) - basswood (Tilia americana) with white pine (Pinus strobus) overstory, respectively). Ohmann (1968)used distance measures to detect pattern and Sudia (1960) used distance, plotless, and quadrat methods and compared the methods. In the current study circular plots of several sizes were used for pattern analysis.

Mapping

All live trees 1.0 inches (2.54 cm) and larger d.b.h. (diameter breast high) on an area of about 3.06 acres (1.24 ha) were mapped using a plane table and alidade; the species and diameter of each tree, as well as its position, were recorded. The mapping area was defined to include pre-existing permanent study plots; the resulting map was trapezoidal in shape, 492 feet (150 m)(along an assumed moisture gradient from the edge of a small swamp to an area of higher elevation) by 246 to 340.2 feet (75 to 90 m), at a scale of 1:100. No vertical relief was mapped as there were only slight elevational changes. The accuracy of the map is about 1.6 feet (0.5 m) on the ground, or 0.016 feet (0.5 cm) on the map.

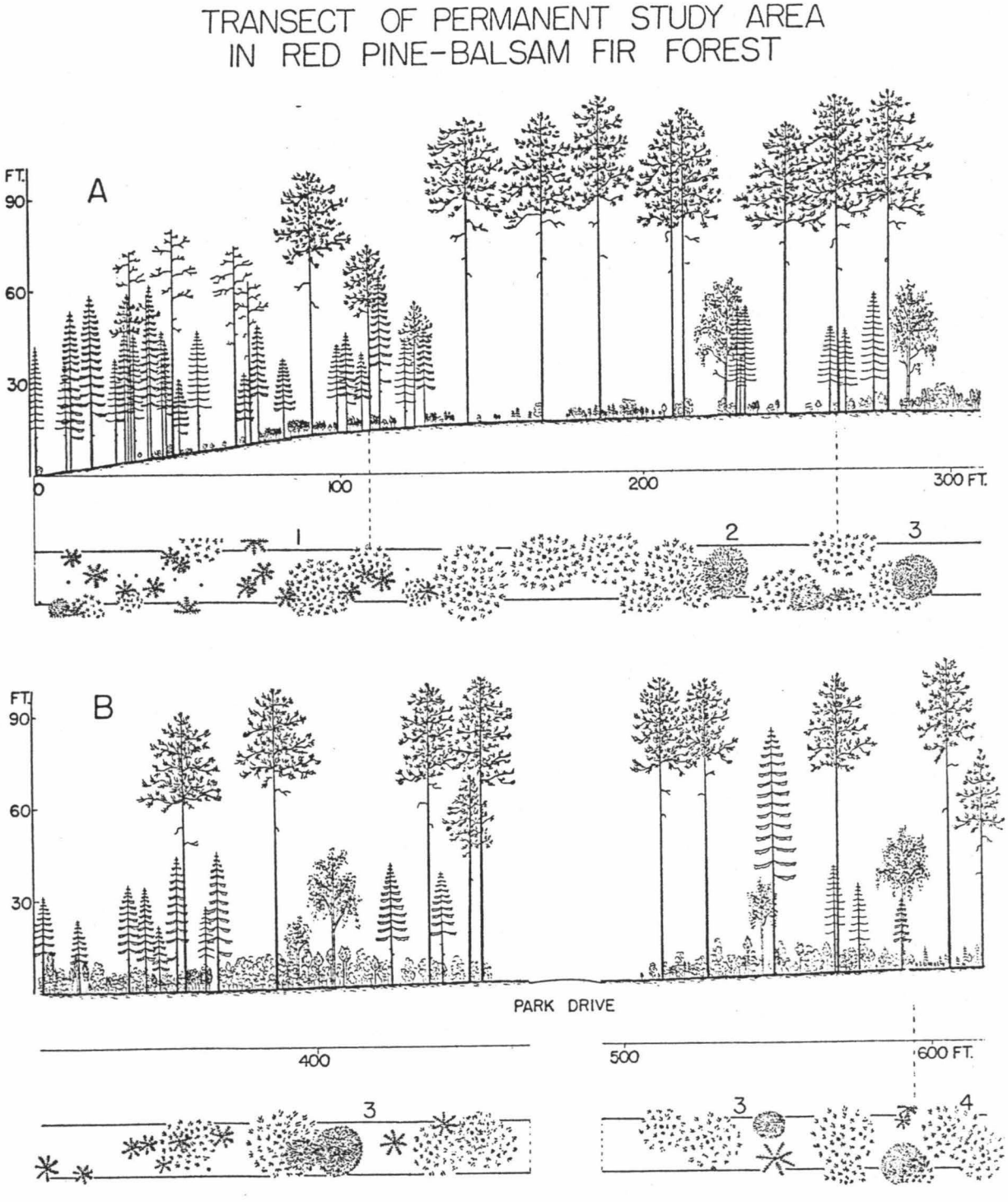

Figure 1. Relative physiognomy of the mapped stand showing red pine (Pinus resinosa), balsam fir (Abies balsamea) and paper birch (Betula papyrifera) (from Kurmis, 1969). Type location: Dense Balsam Fir 0-130, Thinned 130-230', Upland 230' - Park Drive.

The mapped area included three different "types" of vegetation, which were oriented perpendicular to the moisture gradient. Figure 1 shows the positions of these areas and a comparative picture of the type physiognomy (from Kurmis, 1969).

Sampling and Analysis

On the map, an area 60 x 130 cm was delimited; this size was chosen to provide a coordinate sampling grid free from possible map edge inaccuracies. This sampling area was then defined to be a population, and whose parameters could be determined. Table 1 presents the areal size of the three types of vegetation in the sampling area.

Table 1. Areal distribution of types.

Type	Area		% of Total Area
	Acres	Hectares	
Entire Sample Area	1.93	0.78	100
Dense Balsam Fir	0.59	0.238	30.5
Thinned	0.48	0.196	25.1
Upland	0.85	0.346	44.4

The map was sampled by two procedures. The first procedure, a systematic sample, was used to test the homogeneity of the types and the sample area. In each type, 128 contiguous plots were laid out in an approximately rectangular order, oriented parallel to the moisture gradient (east and west). The group of 128 plots of each type were divided into half (north and south direction, 64 plots in each of two samples per type) and then into eighths (each half into quarters), thirty-seconds (each eighth into quarters), and finally each plot taken as a sample, or one hundred twenty-eighths (each thirty-second into quarters). Figure 2 shows diagrammatically how

this divisive sampling procedure was done. At each stage of division a Chi-square test of homogeneity was made on the number of trees and the basal area per sample. Species with less than 0.1 square foot basal area per acre (0.023 m^2/ha) were not included in this sampling procedure.

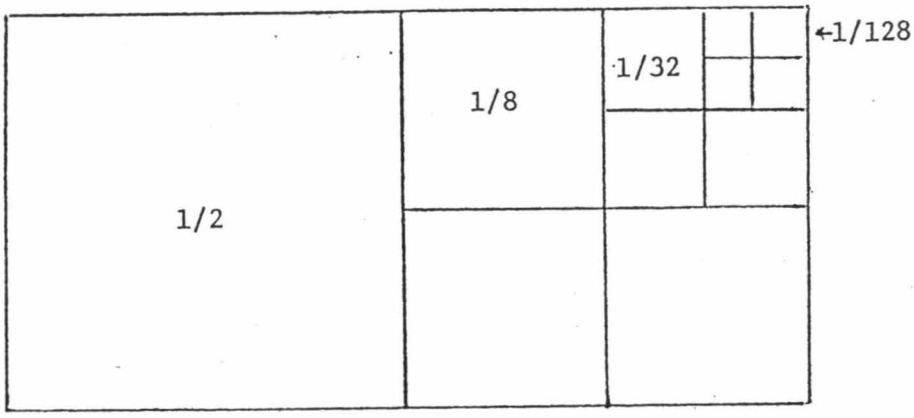

Figure 2. Schematic of the divisive sampling procedure for testing homogeneity.

The other sampling procedure consisted of a series of randomly located, nested plots of 4, 8, 16, 32, 64, and 128 m^2. At each of the 240 randomly located points, plots 4, 8, and 16 m^2 were used, while at each second point the 32 m^2 plot, at each fourth point 64 m^2 plot, and at each eighth point the 128 m^2 plot were used. This sampling procedure yielded a total of 240 plots of 4, 8, and 16 m^2 size, 120 plots of 32 m^2, 60 plots of 64 m^2, and 30 plots of 128 m^2. Table 2 shows descriptive data of this sampling procedure. The species and number of trees in each plot were recorded; the location of the plot in reference to the three types was also recorded. This sampling procedure was designed to aid in the determination of the pattern of the vegetation, both as to species

Table 2. Descriptive data of the random sampling procedure.

Plot Size M^2	No. of Plots	Proportion of population (all types) sampled
4	240	12.3 %
4	120	6.25%
8	240	24.6 %
8	120	12.3 %
16	240	49.2 %
16	120	24.6 %
32	120	49.2 %
64	60	49.2 %
128	30	49.2 %

and for each type. The measure used was similar to variance: mean ratio tests as described by Greig-Smith (1964). In the present study an "Index of Contagion" was used, which is the variance/mean (V/M). This function is distributed Chi-square/degrees of freedom if the population is random and was sampled randomly. Thus, significance of V/M was tested by the use of Chi-square/degrees of freedom tabled values. The variance and mean statistics were computed in the normal manner. An alternative method for the computation of the variance, if a Chi-square test (for homogeneity or some other test) is also being used, is given in Appendix I.

When the ratio of the variance: mean is obtained, a value of 1 indicates the population sampled is random (Poisson); if the value is less than one the population is tending toward regularity, and if greater than 1 the population is more contagious. The Index of Contagion test utilizes directly, with no further computations, the variance: mean ratio and thus the test values are kept in the perspective as to their relationship to the random distribution. These properties are the differences between the current test and those described by Greig-Smith (1964).

III. Results and Discussion

Area Description and Maps

The vegetation overstory of the mapped area consists of two-layers, with predominantly red pine (Pinus resinosa) in the upper layer and balsam fir (Abies balsamea) in the lower layer. The map (Appendix II) lists all tree species present and shows their distribution and diameters; an outline of the sampling area is also shown. Appendix III consists of maps showing the distribution of individual tree species on the sampling area; the nature of these distributions will be discussed later. Table 3 is a listing of diameter-height relationships found on the mapped area (data from unpublished records of E.V. Bakuzis, on which field checks were made), and Table 4 presents a general age distribution of selected species on the area (data also from unpublished records of E.V. Bakuzis). These data show that the red pine overstory was established over the mapped area before the current stand of balsam fir was established. Since red pine is not adapted to regeneration under low light conditions (Fowells, 1965), there probably was not very much, if any, balsam fir (or other tree or shrub cover) present when the red pine became established. The distribution of red pine over the sample area (Map RP, Appendix III) does not show any major visual differences and it therefore can be concluded that if there were any factors influencing the distribution of red pine, they were approximately the same over the area during the periods of red pine establishment.

Table 3. Tree d.b.h. - height relationships on the mapped area (from unpublished data of E.V. Bakuzis).

Type	Species	d.b.h. (inches)	height (feet)
Dense balsam fir	balsam fir	2	25-33
		3	36-40
		4	31
		5	42-48
		6	50
		6.5	58
	red pine	13.5	75
		14.0	80-86
		14.5	82
		18.5	78
		19	103-107
		24	117
	paper birch	2.0	33
		2.5	32
		4.5	43
		6.5	67
	white pine	7.5	70
	white spruce	8	63
Thinned	balsam fir	(Area thinned of balsam fir)	
	red pine	7	80
		13	77
		25	120
	paper birch	3.0	33
		7.5	67
	white pine	(No data)	
	white spruce	8	67
		20	103
Upland	balsam fir	2	17
		2.5	20
		3	28
		4	32-35
		7	65-73
	red pine	17.8	97
		18.6	103
		20.4	92
		21.0	117
		22.0	97-108
	paper birch	4.5	35
	white pine	6.3	43
		8.2	57
		10.0	67
		17.0	97
	white spruce	12	74

Table 4. Age data of selected species on the mapped area as of 1967 (from unpublished data of E.V. Bakuzis).

	Approximate Age (years)			
	balsam fir	red pine	paper birch	white pine
Dense balsam fir	50-80	155-170	< 90	
Thinned	<35	130-255		80-160
Upland	55-70	125-255		

The maps of balsam fir (Maps BF a, b, Appendix III), however, show appreciable differences in distribution of that species. The height and age data, as well as the mapped trees and diameters, also show these differences. The type referred to as "thinned" was cleared of all balsam fir about 1933 (established from unpublished records, by E.V. Bakuzis). Since that time balsam fir has become reestablished on the area, but of course it is currently of small sizes.

Since it is not the purpose of this paper to discuss causes and effects of population distributions, no further discussion of this aspect of the populations will be given. Alternatively, the results of the sampling procedures and a discussion of the current distributions will be given.

Population parameters

The sampling area (as delimited on the map, Appendix II) was systematically sampled (by a 100% sample) to determine the population parameters, as presented in Table 5. Parameters for the total sampling area, as well as for each type, are given.

Systematic Sampling

The results of the tests of homogeneity of the types, as determined by the systematic sampling procedure, are given in Table 6. These results indicate some of the difficulties in sampling with a given plot size and in systematic sampling. For example, when the

Table-5. Population parameters, the sampled area being the population (Species abbrevia here are given in Appendix II).

Type	Species	BF	RP	PB	WP	WS	MM	RM	BA
Total Area	Trees	399	137	95	10	36	11	10	16
	BA (ft^2)	43.6728	277.5005	12.1363	5.4818	2.5536	.1100	.1651	.1260
	BA (m^2)	4.0573	25.7806	1.1275	.5093	.2372	.0102	.0153	.0117
	Trees/A	207.01	71.08	49.29	5.19	18.68	5.71	5.19	8.30
	Trees/Ha	511.54	175.64	121.79	12.82	46.15	14.10	12.82	20.51
	BA ft^2/A	22.6588	143.9756	6.2967	2.8441	1.3249	.0571	.0857	.0654
	BA m^2/Ha	5.2017	33.0520	1.4455	.6529	.3041	.0131	.0196	.0150
Dense BF	Trees	246	37	24	1				
	BA (ft^2)	32.8465	57.9376	4.8154	.3318				
	BA (m^2)	3.0515	5.3826	.4474	.0308				
	Trees/A	418.29	62.91	40.81	1.70				
	Trees/Ha	1033.61	155.46	100.84	4.20				
	BA ft^2/A	55.8509	98.5148	8.1879	.5642				
	BA m^2/Ha	12.8214	22.6160	1.8798	.1294				
Thinned	Trees	75	39	23	3	35			
	BA (ft^2)	.8773	92.8402	1.9571	2.2719	1.4538			
	BA (m^2)	.0815	8.6251	.1812	.2111	.1351			
	Trees/A	154.85	80.52	47.49	6.19	72.27			
	Trees/Ha	382.65	198.98	117.35	15.31	178.57			
	BA ft^2/A	1.8114	191.6890	4.0409	4.6908	3.0017			
	BA m^2/Ha	.4158	44.0056	.9245	1.0770	.6893			
Upland	Trees	78	61	48	6	1	11	10	16
	BA (ft^2)	9.9490	126.7227	5.3638	2.8781	1.0998	.1100	.1651	.1260
	BA (m^2)	.9243	11.7729	.4983	.2674	.1022	.0102	.0153	.0117
	Trees/A	91.23	71.35	56.14	7.02	1.17	12.87	11.70	18.71
	Trees/Ha	225.43	176.30	138.73	17.34	2.89	31.79	28.90	46.24
	BA ft^2/A	11.6364	148.2161	6.2736	3.3663	1.2863	.1287	.1931	.1474
	BA m^2/Ha	2.6714	34.0257	1.4402	.7728	.2954	.0295	.0442	.0338

es abbreviations used

BA	AE	PC	CC	RO	BO	Total	% of Total	% of sampling area within type
16	2	6	1	2	2	727	100	100
.1260	.0330	.0648	.0092	.0110	.1554	342.0195	100	
.0117	.0031	.0060	.0009	.0010	.0144	31.7747		
8.30	1.04	3.11	.52	1.04	1.04	377.19		
20.51	2.56	7.69	1.28	2.56	2.56	932.05		
.0654	.0171	.0336	.0048	.0057	.0806	177.4500		
.0150	.0040	.0077	.0012	.0013	.0185	40.7368		
						308	42.4	30.5
						95.9313	28.0	
						8.9123		
						523.71		
						1294.12		
						163.1177		
						37.4466		
		3				178	24.5	25.1
		.0376				99.4379	29.1	
		.0035				9.2381		
		6.19				367.52		
		15.31				908.16		
		.0776				205.3114		
		.0179				47.1332		
16	2	3	1	2	2	241	33.1	44.4
.1260	.0330	.0272	.0092	.0110	.1554	146.6503	42.9	
.0117	.0031	.0025	.0009	.0010	.0144	13.6243		
18.71	2.34	3.51	1.17	2.34	2.34	281.88		
46.24	5.78	8.67	2.89	5.78	5.78	696.53		
.1474	.0386	.0318	.0108	.0129	.1818	171.5237		
.0338	.0090	.0072	.0026	.0029	.0416	39.3765		

Table 6. The results of the tests for homogeneity of types by the systematic sampling procedure; plot size is the amount of total sample area. (Type abbreviations: DBF = Dense balsam fir, Th = Thinned, Up = Upland.)

Quantity Measured	Type	All Species					Balsam Fir					Red Pine		
		Total Amt. in Sample	Plot Size				Total Amt. in Sample	Plot Size				Total Amt. in Sample	Plot	
			1/2	1/8	1/32	1/128		1/2	1/8	1/32	1/128		1/2	1/8
No. of Trees														
	DBF	120	NS	*	NS	NS	43	NS	NS	NS	NS	29	NS	NS
	Th	170	***	***	***	*	73	***	***	***	***	38	NS	NS
	Up	269	*	NS	NS	NS	216	NS	NS	NS	NS	32	NS	NS
Basal Area (sq.ft.)														
	DBF	71.2759	NS	NS	**	***	5.4262	NS	NS	NS	NS	63.9399	NS	NS
	Th	96.8171	NS	NS	***	***	.7540	NS	NS	NS	NS	87.1929	NS	NS
	Up	88.3240	NS	NS	***	***	29.6159	NS	NS	NS	NS	54.1175	*	NS

H_o: Sample areas have equal No. of trees.

H_o: Sample areas have equal tree basal area

NS = Non-significant

* = 95% significance level
** = 99% significance level
*** = 99.5% significance level

)f types by the systematic sampling procedure;
:ea. (Type abbreviations: DBF = Dense balsam

Balsam Fir					Red Pine					Paper Birch				
il Amt. Sample	Plot Size				Total Amt. in Sample	Plot Size				Total Amt. in Sample	Plot Size			
	1/2	1/8	1/32	1/128		1/2	1/8	1/32	1/128		1/2	1/8	1/32	1/128
3	NS	NS	NS	NS	29	NS	NS	NS	NS	20	***	***	***	NS
3	***	***	***	***	38	NS	NS	NS	NS	23	NS	*	***	***
5	NS	NS	NS	NS	32	NS	NS	NS	NS	21	***	***	***	NS
5.4262	NS	NS	NS	NS	63.9399	NS	NS	***	***	2.3771	NS	NS	NS	NS
.7540	NS	NS	NS	NS	87.1929	NS	NS	***	***	1.7492	NS	NS	NS	NS
9.6159	NS	NS	NS	NS	54.1175	*	NS	***	***	4.2918	NS	*	NS	NS

s.

area

homogeneity of basal area for all species was tested, it was found that when the sample area was divided in halves or eighths, there was a non-significant test of homogeneity, but when smaller plots were taken (thirty-seconds or one hundred twenty-eighths) the tests indicated the sample plots were very much different. Thus the smaller plot sizes were perhaps too small or the larger plot sizes were too large to indicate the pattern; in a systematic sample of this type, a decision in either direction would be dubious. This type of analysis was worthwhile, however, for testing the homogeneity of the following samples:

1. Distribution of trees within types for
   a) All species
      1) Dense balsam fir type (Homogeneous)
      2) Thinned type (Non-homogeneous)
      3) Upland type (Homogeneous)
   b) Balsam fir
      1) Dense balsam fir type (Homogeneous)
      2) Thinned type (Non-homogeneous)
      3) Upland type (Homogeneous)
   c) Red pine
      1) All types (Homogeneous)

2. Distribution of basal area within types for
   a) Balsam fir
      All types (Homogeneous)
   b) Paper birch
      All types (Homogeneous, or very nearly so)

This type of sampling procedure indicates that there is randomness present in the populations, but the degree of randomness cannot be determined.

The homogeneity among the three types was also tested; the results of these tests (Table 7) indicate that the types were different as to the total number of trees (all species), total numbers of balsam fir, and total basal area of balsam fir, but not the other quantities.

This systematic sampling procedure shows that the randomness or non-randomness of distributions may be tested, but not the degree of departure from non-randomness. For that reason the following sampling procedure was conducted.

Table 7. Comparison of types using the total area sampled in each type, as the basis for judgment.

Quantity Measured	All Species		Balsam Fir		Red Pine		Paper Birch	
	Total Amt. in Sample	Test	Total Amt. in Sample	Test	Total Amt. in Sample	Test	Total Amt. in Sample	Test
No. trees	559	***	332	***	99	NS	64	NS
Basal Area (sq. ft.)	256.4170	NS	35.7961	***	205.2503	NS	8.4181	NS

H_o: Types similar as to characteristic
NS = Non-significant
*** = 99.5% significance level

Random Sampling

The results of the tests of the Index of Contagion are given in Table 8. Those values of variance/mean (V/M) which are at or near one indicate that the population is random for the size of plot used; those values of V/M which are at or near one for all the different size plots indicate the population is randomly distributed. It is believed that this sampling procedure was the most valuable in this study and will therefore be discussed in detail.

It is important to recall that the sample area was not sampled by types, but instead the sample was post stratified. Thus the number of plots in each type was dependent on the size of the type.

Over the total sample area it can be concluded that the sampling procedure indicates non-randomness of all species taken together. The loss of randomness is detected by the larger plot sizes (16 - 128M^2) and the sizes of clumps are in this size bracket (since the V/M ratio is greater than 1). The factor contributing to this loss (or lack) of randomness is probably due to the distribution of the balsam fir being non-random (as indicated by the larger plot sizes). Thus it appears that the balsam fir, over the whole sample area, has some non-random clumpings in the 16 -128M^2 size bracket.

The sampling methods indicate that the upland type consists of a random distribution of all trees and of the individual species of balsam fir, red pine, and paper birch. Since the trees in this type are of many different ages, species, and sizes, it appears that communities, at least in the overstory, tend to randomize themselves when relatively undisturbed (as the upland type was).

The tests of the thinned type show results similar to those found for the total sample area. The balsam fir population distribution is quite clumped (probably due to seed dispersal characteristics), which also reflects in the

Table 8. Results of random sampling. (Species abbreviations are given in Appendix II.)

Type	Species		PLOT SIZE $4M^2$ Var/M	Test	$4M^2$ Var/M	Test	$8M^2$ Var/M	Test	$8M^2$ Var/M	Test	16 Var/M	Test
Total Area		No. Plots	240		120		240		120		240	
		% of type sampled	12.3		6.2		24.6		12.3		49.2	
	All		1.061		.938		.996		.884		1.121	
	BF		1.127		1.016		1.137		.975		1.389	***
	RP		.925		.948		.848		.812		.907	
	PB		1.081		.960		1.065		.848		.973	
Upland		No. Plots	106		54		106		54		105	
		% of type sampled	12.3		6.2		12.3		6.2		24.3	
	All		.668		1.046		1.004		.982		.963	
	BF		.957		.973		.886		.910		.801	
	RP		.947		.946		.827		.793		.790	
	PB		1.087		.925		.953		.793		.847	
Thinned		No. Plots	66		33		66		33		68	
		% of type sampled	13.5		6.7		13.5		6.7		55.5	
	All		1.128		.942		1.068		.951		1.245	
	BF		1.405	*	1.283		1.579	*	1.187		1.595	*
	RP		.934		.967		.844		.811		.799	
	PB		1.000		---		1.477	*	.934		1.238	
Dense BF		No. Plots	68		33		68		33		67	
		% of type sampled	11.4		5.5		11.4		5.5		45.0	
	All		.926		.827		.870		.713		1.025	
	BF		.813		.781		.760		.670		.840	
	RP		.932		.967		.890		.909		1.261	
	PB		.977		.967		.886		.909		.974	

Test: blank = No difference, i.e., Accept H_o: Var. = Mean

* = 95% si
** = 99% si
*** = 99.9%

are given in Appendix II.)

PLOT SIZE

$8M^2$		$16M^2$		$32M^2$		$64M^2$		$128M^2$	
Test	Var/M	Test	Var/M	Test	Var/M	Test	Var/M	Test	Var/M	Test
	120		240		120		120		60		30	
	12.3		49.2		24.6		49.2		49.2		49.2	
	.884		1.121		1.425	**	1.469	**	1.478	*	2.555	***
	.975		1.389	***	1.476	**	2.156	***	2.992	***	5.047	***
	.812		.907		.959		1.024		1.027		.603	
	.848		.973		.835		1.213		1.177		1.259	
	54		105		53		53		29		14	
	6.2		24.3		12.3		49.0		53.6		51.8	
	.982		.963		1.061		.869		.643		.906	
	.910		.801		.866		.707		.997		.851	
	.793		.790		.785		.964		.714		.617	
	.793		.847		.763		.982		.558		1.049	
	33		68		35		35		20		13	
5	6.7		55.5		28.6		57.1		65.3		54.4	
	.951		1.245		1.569	*	1.576	*	2.037	**	2.830	***
*	1.187		1.595	*	1.981	***	1.657	*	3.316	***	3.239	***
	.811		.799		.848		.666		.935		.362	*
*	.934		1.238		.881		.825		1.516		1.348	
	33		67		32		32		11		3	
4	5.5		45.0		21.5		43.0		29.6		16.1	
	.713		1.025		.843		1.372		1.070		4.663	**
	.670		.840		.484	*	1.483		.888		7.182	***
	.909		1.261		1.427		1.184		1.000		2.33	
	.909		.974		.872		1.758	*	1.670		2.000	

* = 95% significance level
** = 99% significance level
*** = 99.9% significance level

distribution of the population of all species on the area. Red pine and paper birch (except in one instance which may be attributed to random error) are randomly (or nearly so) distributed. The larger values of V/M for the paper birch indicates that the population distribution is tending away from randomness toward contagiousness, but not too much as indicated by the non-significant tests. Again, the larger plot sizes detected the clumping of the balsam fir.

The dense balsam fir type tests show nearly the same results as those for the upland type. The one instance of the paper birch showing a contagious distribution may be attributed to error, but it is more likely that there is some clumping; in general, the V/M ratios are increasing with plot size, in the larger sizes. The distribution of balsam fir again affects the distribution pattern of all species, as indicated at the $128M^2$ plot size. The significant test at the $16M^2$ plot size may be disregarded as experimental error as the test of the 67 plots of the same size (compared to 32 plots) shows a value much closer to 1. The high significance value of the balsam fir at the $128M^2$ plot size does not have much weight (nor does the same test for all species) as the test is based only on the three replications in the type; there may be some contagion of the balsam fir as indicated by the test of this plot size, but the test for it based solely on three replications may be questioned. It may be concluded, however, that the population distribution of the balsam fir is nearly random, based on the tests of all other plot sizes.

Perhaps the single most important result of this sampling procedure and method of testing is the emphasis that one cannot simply use a given plot size to test the randomness of a population, unless the population is known to be random or nearly so. Also, only a small percentage of the total area need be sampled, under the same conditions.

Comparison of Sampling Methods

The results of the two sampling procedures are nearly comparable, except that the random procedure also provides information on the degree of departure from randomness. In the discussion of the results of the systematic procedure, the following points were emphasized:

Homogeneous (= Random)

All species in: Dense balsam fir type

Upland type

Balsam fir in: Dense balsam fir type

Upland type

Red pine in: All types

Non-Homogeneous (= Non-random)

All species in: Thinned type

Balsam fir in: Thinned type

The random procedure results emphasized these points and others in addition. Thus it is concluded that the random procedure is the better of the two methods.

IV. Summary of Conclusions

When examining a population to determine its distribution, a random selection of sample points and the use of a series of nested plots appears to be a useful method. By calculating the variance and mean of the occurrence of the individuals of the population, an Index of Contagion (Variance/Mean) may be calculated whose value represents the randomness (V/M = 1) or departure from randomness (V/M<1, V/M>1) of the population's distribution. Whether or not the departures from randomness are significant may be tested by the use of a Chi-square/degrees of freedom table. In the current study populations were found to be random (red pine, balsam fir (in one type),

paper birch, and all species taken together (in some types)) and contagious (balsam fir (in two types), and all species taken together (in some types)).

Literature Cited

Bakuzis, Egolfs V. Unpublished records.

Fowells, H.A. 1965. Silvics of Forest Trees of the United States. Agriculture Handbook No. 271. USDA, Washington, D.C. 762 pp.

Greig-Smith, P. 1964. Quantitative Plant Ecology. Butterworths, London. 256 pp.

Kurmis, Vilis. 1969. Dynamics of advance reproduction in upland forest communities in Itasca State Park, Minnesota. Ph.D. Dissertation, University of Minnesota. 166 pp.

Ohmann, Lewis F. 1968. Random tree distribution in jack pine plains near Itasca Park, Minnesota. Bull. Torrey Bot. Club 95(1): 28-31.

Sudia, Theodore W. 1960. Frequency calculations from several different forest ecological sampling methods. Ohio Jour. Sci. 60(2): 100-105.

Appendix I.

An Alternative to Calculation of Variance/Mean

If the desired test is to measure the departure of populations from a random (Poisson) distribution, tabulated values of the individual terms of the Poisson formula

$$f(x) = \frac{M^x e^{-M}}{x!}, \quad M > 0, \quad x = 0, 1, 2, \ldots$$

can be used as the expected value in the calculation of

$$X^2 = \Sigma \frac{(\text{observed value} - \text{expected value})^2}{\text{expected value}}$$

$$= \frac{\text{Sum of Squares}}{\text{mean}} = \frac{s^2(n)}{\text{mean}}$$

which yields the formula

$$\frac{X^2}{n} = \frac{s^2}{\text{mean}} = \frac{\text{Variance}}{\text{Mean}}$$

where n is the number of degrees of freedom. The observed values are the actual number of trees per plot.

Appendix II

Map of the red pine - balsam fir stand.

The area delimited by the red line is the sampling area.

Species recorded on the map:

Species	Common Name	Abbreviation
Abies balsamea	balsam fir	BF
Acer rubrum	red maple	RM
Acer saccharum	sugar maple	SM
Acer spicatum	mountain maple	MM
Betula papyrifera	paper birch	PB
Fraxinus nigra	black ash	BA
Picea glauca	white spruce	WS
Pinus resinosa	red pine	RP
Pinus strobus	white pine	WP
Prunus pennsylvanica	pin cherry	PC
Prunus virginiana	choke cherry	CC
Quercus macrocarpa	bur oak	BO
Quercus rubra	red oak	RO
Ulmus americana	American elm	AE

Appendix III

Individual species distributions in the sampling area.

Maps included:

Map P.	Location of thinned area and permanent study points in the sampling area.
Map A.	Distribution of all species of trees ≥1" d.b.h. in the sampling area.
Map RP.	Red pine distribution in the sampling area.
Map BF, a.	Balsam fir (≥2" d.b.h.) distribution in the sampling area.
Map BF, b.	Balsam fir (<2" d.b.h.) distribution in the sampling area.
Map PB.	Paper birch distribution in the sampling area.
Map WP, WS.	White pine and white spruce distributions in the sampling area.
Map HW.	Distributions of hardwoods other than paper birch.